

Interacting HDE and NADE in Brans-Dicke Chameleon Cosmology

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Motivated by the recent work of one of us [1], we generalize this work to the case where the pressureless dark matter and the holographic dark energy do not conserve separately but interact with each other. We investigate the cosmological applications of interacting holographic dark energy in Brans-Dicke theory with chameleon scalar field which is non-minimally coupled to the matter field. We find out that in this model the phantom crossing can be constructed if the model parameters are chosen suitably. We also perform the study for the new agegraphic dark energy model and calculate some relevant cosmological parameters and their evolution.

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I. INTRODUCTION

Among various scenarios to explain the acceleration of the universe expansion, the holographic dark energy (HDE) and agegraphic dark energy (ADE) models have got a lot of enthusiasm recently. These models are originated from some considerations of the features of the quantum theory of gravity. That is to say, the HDE and ADE models possess some significant features of quantum gravity. Although a complete theory of quantum gravity has not established yet today, we still can make some attempts to investigate the nature of dark energy according to some principles of quantum gravity. The former is motivated from the holographic principle [2, 3]. It was shown in [4] that in quantum field theory, the UV cutoff Λ should be related to the IR cutoff L due to limit set by forming a black hole. If $\rho_D = \Lambda^4$ is the vacuum energy density caused by UV cutoff, the total energy of size L should not exceed the mass of the system-size black hole:

$$E_D \leq E_{BH} \rightarrow L^3 \rho_D \leq m_p^2 L. \quad (1)$$

If the largest cutoff L is taken for saturating this inequality, we get the energy density of HDE as

$$\rho_D = \frac{3c^2 m_p^2}{L^2} = \frac{3c^2}{8\pi G L^2}. \quad (2)$$

The HDE is thoroughly investigated in the literature in various ways (see e.g [5] and references therein). The later (ADE) model assumes that the observed dark energy comes from the space-time and matter field fluctuations in the universe. Following the line of quantum fluctuations of spacetime, Karolyhazy et al. [6] discussed that the distance t in Minkowski spacetime cannot be known to a better accuracy than $\delta t = \beta t_p^{2/3} t^{1/3}$ where β is a dimensionless constant of order unity. Based on Karolyhazy relation, Sasakura [7] discussed that the energy density of metric fluctuations of the Minkowski spacetime is given by (see also [8])

$$\rho_D \sim \frac{1}{t_p^2 t^2} \sim \frac{m_p^2}{t^2}, \quad (3)$$

where t_p is the reduced Planck time and t is a proper time scale. On these basis, Cai [9] proposed the energy density of the original ADE in the form

$$\rho_D = \frac{3n^2 m_p^2}{T^2}, \quad (4)$$

where T is the age of the universe. Since the original ADE model suffers from the difficulty to describe the matter-dominated epoch, the new ADE (NADE) model was proposed by Wei and Cai [10], while the time scale was chosen to be the conformal time instead of the age of the universe.

The ADE models have arisen a lot of enthusiasm recently and have examined and studied in ample detail [11–14].

It is also of great interest to analyze these models in the framework of Brans-Dicke (BD) gravity. In recent years the BD theory of gravity got a new impetus as it arises naturally as the low energy limit of many theories of quantum gravity such as superstring theory or Kaluza-Klein theory. The motivation for studying these models in the BD theory comes from the fact that both HDE and ADE models belong to a dynamical cosmological constant, therefore we need a dynamical frame to accommodate them instead of Einstein gravity. The investigation on the HDE and ADE models in the framework of BD cosmology, have been carried out in [15–18]. In the present work, we consider a BD theory in which there is a non-minimal coupling between the scalar field and the matter field. Thus the action and the field equations are modified due to the coupling of the scalar field with the matter. This kind of scalar field usually called “chameleon” field in the literature [19]. This is due to the fact that the physical properties of the field, such as its mass, depend sensitively on the environment. Moreover, in regions of high density, the chameleon blends with its environment and becomes essentially invisible to searches for Equivalence Principle violation and fifth force [19]. Further more, it was shown [19, 20] that all existing constraints from planetary orbits, such as those from lunar laser ranging, are easily satisfied in the presence of chameleon field. The reason is that the chameleon-mediated force between two large objects, such as the Earth and the Sun, is much weaker than one would naively expect. In particular, it was shown [20] that the deviations from Newtonian gravity due to the chameleon field of the Earth are suppressed by nine orders of magnitude by the thin-shell effect. Other studies on the chameleon gravity have been carried out in [21]. Our work differs from that of Ref. [17] in that we assume a non-minimal coupling between the scalar field and the matter field. It also differs from that of Ref. [1], in that we assume the pressureless dark matter and dark energy do not conserve separately but interact with each other, while the author of [1] assumes that the dark components do not interact with each other.

II. HDE IN BD THEORY WITH CHAMELEON SCALAR FIELD

We begin with the BD chameleon theory in which the scalar field is coupled non-minimally to the matter field via the action [22]

$$S = \int d^4x \sqrt{-g} \left(\phi R - \frac{\omega}{\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) + f(\phi) L_m \right), \quad (5)$$

where R is the Ricci scalar curvature, ϕ is the BD scalar field with a potential $V(\phi)$. The chameleon field ϕ is non-minimally coupled to gravity, ω is the dimensionless BD parameter. The last term in the action indicates the interaction between the matter Lagrangian L_m and some arbitrary function $f(\phi)$ of the BD scalar field. In the limiting case $f(\phi) = 1$, we obtain the standard BD theory.

The gravitational field equations derived from the action (5) with respect to the metric is

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{f(\phi)}{\phi}T_{\mu\nu} + \frac{\omega}{\phi^2}\left(\phi_\mu\phi_\nu - \frac{1}{2}g_{\mu\nu}\phi^\alpha\phi_\alpha\right) + \frac{1}{\phi}[\phi_{\mu;\nu} - g_{\mu\nu}\square\phi] - g_{\mu\nu}\frac{V(\phi)}{2\phi}. \quad (6)$$

where $T_{\mu\nu}$ represents the stress-energy tensor for the fluid filling the spacetime which is represented by the perfect fluid

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}, \quad (7)$$

where ρ and p are the energy density and pressure of the perfect fluid which we assume to be a mixture of matter and dark energy. Also u^μ is the four-vector velocity of the fluid satisfying $u^\mu u_\mu = -1$. The Klein-Gordon equation (or the wave equation) for the scalar field is

$$\square\phi = \frac{T}{2\omega + 3}\left(f - \frac{1}{2}\phi f_{,\phi}\right) + \frac{1}{2\omega + 3}(\phi V_{,\phi} - 2V), \quad (8)$$

where T is the trace of (7). The homogeneous and isotropic Friedmann-Robertson-Walker (FRW) universe is described by the metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2\right), \quad (9)$$

where $a(t)$ is the scale factor, and $k = -1, 0, +1$ corresponds to open, flat, and closed universes, respectively. Variation of action (5) with respect to metric (9) for a universe filled with dust and HDE yields the following field equations

$$H^2 + \frac{k}{a^2} - \frac{\omega}{6}\frac{\dot{\phi}^2}{\phi^2} + H\frac{\dot{\phi}}{\phi} = \frac{f(\phi)}{3\phi}(\rho_M + \rho_D) + \frac{V(\phi)}{6\phi}, \quad (10)$$

$$2\frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} + 2H\frac{\dot{\phi}}{\phi} + \frac{\ddot{\phi}}{\phi} = -\frac{p_D}{\phi} + \frac{V(\phi)}{2\phi}, \quad (11)$$

where $H = \dot{a}/a$ is the Hubble parameter, ρ_D , p_D and ρ_M are, respectively, the dark energy density, dark energy pressure and energy density of dust (dark matter). Here, a dot indicates differentiation with respect to the cosmic time t . The dynamical equation for the scalar field is

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\rho - 3p}{2\omega + 3}\left(f - \frac{1}{2}\phi f_{,\phi}\right) + \frac{2}{2\omega + 3}\left(V - \frac{1}{2}\phi V_{,\phi}\right) = 0. \quad (12)$$

We assume the HDE in the chameleon BD theory has the following form

$$\rho_D = \frac{3c^2\phi}{L^2}. \quad (13)$$

The motivation idea for taking the energy density of HDE in BD theory in the form (13) comes from the fact that in BD theory we have $\phi \propto G^{-1}$. Here the constant $3c^2$ is introduced for later convenience and the radius L is defined as

$$L = ar(t), \quad (14)$$

where the function $r(t)$ can be obtained from the following relation

$$\int_0^{r(t)} \frac{dr}{\sqrt{1 - kr^2}} = \int_0^\infty \frac{dt}{a} = \frac{R_h}{a}. \quad (15)$$

It is important to note that in the non-flat universe the characteristic length which plays the role of the IR-cutoff is the radius L of the event horizon measured on the sphere of the horizon and not the radial size R_h of the horizon. Solving Eq. (16) for the general case of the non-flat FRW universe, we get

$$r(t) = \frac{1}{\sqrt{k}} \sin y, \quad (16)$$

where $y = \sqrt{k}R_h/a$. Now we define the critical energy density, ρ_{cr} , and the energy density of the curvature, ρ_k , as

$$\rho_{\text{cr}} = 3\phi H^2, \quad \rho_k = \frac{3k\phi}{a^2}. \quad (17)$$

As usual, the fractional energy densities are defined as

$$\Omega_M = \frac{\rho_M}{\rho_{\text{cr}}} = \frac{\rho_M}{3\phi H^2}, \quad (18)$$

$$\Omega_k = \frac{\rho_k}{\rho_{\text{cr}}} = \frac{k}{H^2 a^2}, \quad (19)$$

$$\Omega_D = \frac{\rho_D}{\rho_{\text{cr}}} = \frac{c^2}{H^2 L^2}. \quad (20)$$

For latter convenience we rewrite Eq. (20) in the form

$$HL = \frac{c}{\sqrt{\Omega_D}}. \quad (21)$$

Taking derivative with respect to the cosmic time t from Eq. (14) and using Eqs. (16) and (21) we obtain

$$\dot{L} = HL + a\dot{r}(t) = \frac{c}{\sqrt{\Omega_D}} - \cos y. \quad (22)$$

Consider the FRW universe filled with dark energy and pressureless matter which evolves according to their conservation laws

$$\dot{\rho}_D + 3H\rho_D(1 + w_D) = 0, \quad (23)$$

$$\dot{\rho}_M + 3H\rho_M = 0, \quad (24)$$

where w_D is the equation of state parameter of dark energy. At this point our system of equations is not closed and we still have freedom to choose one. We shall assume that BD field can be described as a power law of the scale factor, $\phi \propto a^\alpha$. In principle there is no compelling reason for this choice. However, it has been shown that for small α it leads to consistent results [15]. A case of particular interest is that when α is small whereas ω is high so that the product $\alpha\omega$ results of order unity [15]. This is interesting because local astronomical experiments set a very high lower bound on ω [23]; in particular, the Cassini experiment implies that $\omega > 10^4$ [24, 25]. Taking the derivative with respect to time of relation $\phi \propto a^\alpha$ we get

$$\dot{\phi} = \alpha H \phi, \quad (25)$$

$$\ddot{\phi} = \alpha^2 H^2 \phi + \alpha \phi \dot{H}. \quad (26)$$

Taking the derivative of Eq. (13) with respect to time and using Eqs. (22) and (25) we reach

$$\dot{\rho}_D = H\rho_D \left(\alpha - 2 + 2\frac{\sqrt{\Omega_D}}{c} \cos y \right). \quad (27)$$

Substituting this equation in Eq. (23), we obtain the equation of state parameter

$$w_D = -\frac{1}{3}(\alpha + 1) - \frac{2\sqrt{\Omega_D}}{3c} \cos y. \quad (28)$$

It is important to note that in the limiting case $\alpha = 0$ ($\omega \rightarrow \infty$), the Brans-Dicke scalar field becomes trivial and Eq. (28) reduces to its respective expression in Einstein gravity [3]

$$w_D = -\frac{1}{3} - \frac{2\sqrt{\Omega_D}}{3c} \cos y. \quad (29)$$

We will see that the combination of the Brans-Dicke field and HDE brings rich physics. For $\alpha \geq 0$, w_D is bounded from below by

$$w_D = -\frac{1}{3}(\alpha + 1) - \frac{2\sqrt{\Omega_D}}{3c}. \quad (30)$$

Assuming $\Omega_D = 0.73$ for the present time and choosing $c = 1$ [26], the lower bound becomes $w_D = -\frac{\alpha}{3} - 0.9$. Thus for $\alpha \geq 0.3$ we have $w_D \leq -1$. This implies that the phantom crossing can be constructed in the BD framework. We can also obtain the deceleration parameter

$$q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}, \quad (31)$$

which combined with the Hubble parameter and the dimensionless density parameters form a set of useful parameters for the description of the astrophysical observations. Dividing Eq. (11) by H^2 , and using Eqs. (13), (21), (25) and (26), we find

$$q = \frac{1}{\alpha + 2} \left[(\alpha + 1)^2 + \alpha \left(\frac{\alpha\omega}{2} - 1 \right) + \Omega_k + 3\Omega_D w_D - \frac{3}{2}\Omega_V \right]. \quad (32)$$

where the last term can be understood as a contribution of the potential energy in the total energy density i.e.

$$\Omega_V = \frac{V}{\rho_{\text{cr}}}. \quad (33)$$

Substituting w_D from Eq. (28) in (32), we get

$$q = \frac{1}{\alpha + 2} \left[(\alpha + 1)^2 + \alpha \left(\frac{\alpha\omega}{2} - 1 \right) + \Omega_k - (\alpha + 1)\Omega_D - \frac{2}{c}\Omega_D^{3/2} \cos y - \frac{3}{2}\Omega_V \right]. \quad (34)$$

If we take $\Omega_D = 0.73$ and $\Omega_k \approx 0.01$ for the present time and choosing $c = 1$, $\alpha\omega \approx 1$, $\omega = 10^4$ and $\cos y \simeq 1$, we obtain $q = -0.48$ for the present value of the deceleration parameter which is in good agreement with recent observational results [27].

From equation (12), we can also estimate the mass of the chameleon field. This can be done by calculating the second derivative of the potential function with respect to scalar field [28]. We get

$$m_\phi^2 \equiv V_{,\phi\phi} = \frac{1}{\phi} \left[V_{,\phi} - \frac{\rho - 3p}{2} (f_{,\phi} - \phi f_{,\phi\phi}) \right]. \quad (35)$$

Following previous studies [22, 28], we choose

$$V(\phi) = \frac{M^{4+n}}{\phi^n}, \quad f(\phi) = f_0 e^{b_0 \phi}. \quad (36)$$

Here M , f_0 and b_0 are finite parameters whose values are model dependent. Making use of Eq. (36) in (35), we obtain

$$m_\phi^2 = -\frac{1}{\phi} \left[n \frac{M^{4+n}}{\phi^{n+1}} + \frac{b_0 f_0 e^{b_0 \phi}}{2} (\rho - 3p)(1 - b_0 \phi) \right]. \quad (37)$$

Clearly when $n \rightarrow 0$ (which corresponds to a constant potential), the mass of the scalar field will be dependent on the properties of $f(\phi)$. Moreover if only $\phi = 1/b_0$, the mass is determined by the scalar potential function alone.

III. INTERACTING HDE IN BD THEORY WITH CHAMELEON SCALAR FIELD

In this section we would like to construct a cosmological model based on the BD chameleon field theory of gravity and on the assumption that the dark energy and dark matter do not conserve separately but interact with each other. Taking the interaction into account the continuity

equations becomes

$$\dot{\rho}_D + 3H\rho_D(1 + w_D) = -Q, \quad (38)$$

$$\dot{\rho}_M + 3H\rho_M = Q, \quad (39)$$

where Q is an interaction term which can be an arbitrary function of cosmological parameters like the Hubble parameter and energy densities. The dynamics of interacting dark energy models with different interaction terms have been investigated in [29]. It should be noted that the ideal interaction term must be motivated from the theory of quantum gravity. In the absence of such a theory, we rely on pure dimensional basis for choosing an interaction Q . Hence following [30], we assume $Q = \Gamma\rho_D$ with $\Gamma = 3b^2(1 + r)H$ where $r = \rho_M/\rho_D$ is the ratio of energy densities and b^2 is a coupling constant. Note that $\Gamma > 0$ shows that there is an energy transfer from the dark energy to dark matter. Combining Eqs. (17) and (25) with the first Friedmann equation (10), we can rewrite this equation as

$$1 + \Omega_k = f(\phi)(\Omega_M + \Omega_D) + \Omega_\phi + \frac{1}{2}\Omega_V, \quad (40)$$

where

$$\Omega_\phi = \alpha \left(\frac{\alpha\omega}{6} - 1 \right). \quad (41)$$

Combining Eqs. (27), (40) and (41) with Eq. (38) we find the equation of state parameter of the interacting HDE

$$w_D = -\frac{1}{3}(\alpha + 1) - \frac{2\sqrt{\Omega_D}}{3c} \cos y - \frac{b^2}{f(\phi)\Omega_D} \left[1 + \Omega_k + \alpha \left(1 - \frac{\alpha\omega}{6} \right) - \frac{1}{2}\Omega_V \right]. \quad (42)$$

In the absence of the BD field ($\alpha = 0$, $f(\phi) = 1$, $V(\phi) = 0$), Eq. (42) restores its respective expression in non-flat standard cosmology [31]

$$w_D = -\frac{1}{3} - \frac{2\sqrt{\Omega_D}}{3c} \cos y - \frac{b^2}{\Omega_D} (1 + \Omega_k). \quad (43)$$

Next, we examine the deceleration parameter, $q = -\ddot{a}/(aH^2)$. Substituting w_D from Eq. (42) in Eq. (32), one can easily show

$$q = \frac{1}{\alpha + 2} \left[(\alpha + 1)^2 + \alpha \left(\frac{\alpha\omega}{2} - 1 \right) + \Omega_k - (\alpha + 1)\Omega_D - \frac{2}{c}\Omega_D^{3/2} \cos y - \frac{3}{2}\Omega_V \right. \\ \left. - \frac{3b^2}{f(\phi)} \left(1 + \Omega_k + \alpha \left(1 - \frac{\alpha\omega}{6} \right) - \frac{1}{2}\Omega_V \right) \right]. \quad (44)$$

Comparing Eq. (44) with (34) shows that in the presence of interaction the chameleon function $f(\phi)$ enters explicitly in q expression. This is in contrast to the usual BD theory where q of the interacting HDE model does not depend on the scalar field [17].

Finally we present the equation of motion of the dark energy. Taking the derivative of Eq. (20) and using Eq. (22) and relation $\dot{\Omega}_D = H\Omega'_D$, we find

$$\Omega'_D = 2\Omega_D \left(-\frac{\dot{H}}{H^2} - 1 + \frac{\sqrt{\Omega_D}}{c} \cos y \right), \quad (45)$$

where the dot is the derivative with respect to time and the prime denotes the derivative with respect to $x = \ln a$. Using relation $q = -1 - \frac{\dot{H}}{H^2}$, we have

$$\Omega'_D = 2\Omega_D \left(q + \frac{\sqrt{\Omega_D}}{c} \cos y \right), \quad (46)$$

where q is given by Eq. (44). This equation describes the evolution behavior of the interacting HDE in BD cosmology with chameleon field.

IV. INTERACTING NADE WITH CHAMELEON SCALAR FIELD

The above study can also be performed for the new agegraphic dark energy (NADE) model. In NADE, the infrared cut-off is the conformal time which is defined as

$$\eta = \int \frac{dt}{a} = \int_0^a \frac{da}{Ha^2}. \quad (47)$$

In the framework of BD chameleon scalar field, we assume the following form for the energy density of the NADE

$$\rho_D = \frac{3n^2\phi}{\eta^2}. \quad (48)$$

where the numerical factor $3n^2$ is introduced to parameterize some uncertainties, such as the species of quantum fields in the universe, the effect of curved space-time and so on. The respective fractional energy densities can be written as

$$\Omega_D = \frac{\rho_D}{\rho_{\text{cr}}} = \frac{n^2}{H^2\eta^2}. \quad (49)$$

Differentiating Eq. (48) and using Eqs. (25) and (49) we have

$$\dot{\rho}_D = H\rho_D \left(\alpha - \frac{2}{na} \sqrt{\Omega_D} \right). \quad (50)$$

Substituting this relation in Eq. (38) and using relations (40) and (41), we obtain the equation of state parameter of the interacting NADE

$$w_D = -1 - \frac{1}{3}\alpha + \frac{2}{3na} \sqrt{\Omega_D} - \frac{b^2}{f(\phi)\Omega_D} \left[1 + \Omega_k + \alpha \left(1 - \frac{\alpha\omega}{6} \right) - \frac{1}{2}\Omega_V \right]. \quad (51)$$

When $\alpha = 0$, $f = 1$ and $V = 0$, the BD scalar field becomes trivial and Eq. (51) reduces to its respective expression in NADE in Einstein gravity [12]. From Eq. (51), we see that even in the absence of interaction ($b = 0$), the phantom crossing will take place in the framework of BD theory provided the model parameters are chosen suitably. Indeed in this case ($b = 0$), w_D can cross the phantom divide provided $na\alpha > 2\sqrt{\Omega_D}$. If we take $\Omega_D = 0.73$ and $a = 1$ for the present time, the phantom-like equation of state can be accounted if $n\alpha > 1.7$. For instance, for $n = 4$ and $\alpha = 0.5$, we get $w_D = -1.02$. When the interaction is taken into account the phantom crossing for w_D can be more easily achieved for than when resort to the Einstein field equations is made.

In the context of BD chameleon scalar field the deceleration parameter of interacting NADE is obtained as

$$q = \frac{1}{\alpha + 2} \left[(\alpha + 1)^2 + \alpha \left(\frac{\alpha\omega}{2} - 1 \right) + \Omega_k - (\alpha + 3)\Omega_D + \frac{2}{na}\Omega_D^{3/2} - \frac{3}{2}\Omega_V \right. \\ \left. - \frac{3b^2}{f(\phi)} \left(1 + \Omega_k + \alpha \left(1 - \frac{\alpha\omega}{6} \right) - \frac{1}{2}\Omega_V \right) \right]. \quad (52)$$

While the equation of motion for Ω_D takes the form

$$\Omega'_D = 2\Omega_D \left(1 + q - \frac{\sqrt{\Omega_D}}{na} \right). \quad (53)$$

V. CONCLUSIONS

In this paper, we have considered interacting HDE model in the framework of BD cosmology where the HDE density $\rho_D = 3c^2/(8\pi GL^2)$ is replaced with $\rho_D = 3c^2\phi/L^2$. With this replacement in BD theory, we found that the cosmic acceleration will be more easily achieved for than when the standard HDE is considered. Following the work of [1], we assumed that the scalar field is non-minimally coupled with the matter field via an arbitrary coupling function $f(\phi)$. In principle, the coupling between BD scalar field and matter field should be derived from a theory of quantum gravity. In the absence of such a theory, we have kept our analysis general regardless of the specification of $f(\phi)$. In the present paper, we have extended the work [1] by incorporating the interaction term in the HDE model. An interesting consequence of the present model is that it allows the phantom crossing of the equation of state of dark energy due to the presence of several free parameters. We have also performed the analysis for the NADE model and calculate some relevant cosmological parameters such as the equation of state, deceleration parameter and energy density parameter.

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